Extrapolation for PageRank and Multilinear PageRank

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Abstract

An important problem in web search is to determine the importance of each page. The most popular ranking algorithm is PageRank (used by Google) and developed in 1998 by S. Brin and L. Page [9].

From the mathematical point of view, the calculation of the PageRank vector consists in finding the nonnegative left eigenvector of a matrix corresponding to its dominant eigenvalue 1. To do this, the power method was used. Since this matrix is neither stochastic nor irreducible, the power method has convergence problems. So, the matrix is replaced by a convex combination, depending on a parameter c, with a rank one matrix. Its left principal eigenvector now depends on c, and it is the PageRank vector we are looking for. However, when c is close to 1, the problem is ill-conditioned, and the power method converges slowly.

In 2003, Kamvar et al [7] proposed some acceleration methods. In 2006, and 2008 with C. Brezinski [1, 2], we proposed several different expressions of the PageRank vector and we gave a theoretical justification for the methods given in [7]. We also generalized the Quadratic Extrapolation. Other acceleration results are given for the various ε-algorithms. Another possibility is to use extrapolation methods. The idea was to compute the PageRank vector for several values of c, and then to extrapolate them, by a conveniently chosen rational function, at a point near 1. The choice of this extrapolating function is based on the mathematical expression of the PageRank vector as a function of c.

Recently, the idea of PageRank has been extended to Higher Order Markov Chains by Gleich et al [6]. Although this extension has attractive theoretical properties, it is computationally intractable for problems of large size; hence an approximation of the ideal Higher Order PageRank vector is introduced, called Multilinear PageRank vector.

Its computation can be considerably sped-up using extrapolation techniques, encouraging further theoretical investigation. In particular, with S. Cipolla and F. Tudisco [5], we showed how the sequence generated by two fixed point-type techniques as the SS-HOPM by Kolda and Mayo [8] and the Inner-Outer Method of Gleich et al [6], are accelerated using the The Simplified Topological ε-Algorithm (STEA) in the restarted form [3, 4].

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